*November* 19, 1891.

Mr. JOHN EVANS, D.C.L., LL.D., Treasurer, in the Chair.

Mr. William Anderson and Professor Frederick Orpen Bower were admitted into the Society.

A List of the Presents received was laid on the table, and thanks ordered for them.

In pursuance of the Statutes, notice of the ensuing Anniversary Meeting was given from the Chair.

Sir James Cockle, Mr. F. Galton, and Mr. Stainton were by ballot elected Auditors of the Treasurer's accounts on the part of the Society.

The following Papers were read :—

## I. "The Thermal Emissivity of Thin Wires in Air." By W. E. Ayrton, F.R.S., and H. Kilgour. Received July 2, 1891.

(Abstract.)

In 1884 it was observed experimentally that whereas the electric current required to maintain a *thick* wire of given material, under given conditions, at a given temperature was approximately proportional to the diameter of the wire raised to the power three halves, the current was more nearly proportional to the first power of the diameter if the wire were *thin*. When this difference in the behaviour of a thick and thin wire was first noticed it was regarded as being quite unexpected. But, as pointed out by one of us in the course of a discussion at a meeting of the Royal Society, the unexpected character of the result was due to people having assumed that the loss of heat from radiation and convection per square centimetre of surface per 1° excess temperature was a constant, and independent of the size and shape of the cooling body.

The very valuable investigations that have been made on emissivity by Mr. Macfarlane, Professor Tait, Mr. Crookes, Mr. J. T. Bottomley, and by Mr. Schleiermacher had for their object the determination of the variation of the emissivity with changes of the surface and with change in the density of the gas surrounding the cooling body, but it was not part of these investigations to determine the change in the

emissivity that is produced by change in the shape and size of the cooling body. Indeed, so little has been the attention devoted to the very large change that can be brought about in the value of the emissivity by simply changing the dimensions of the cooling body, that in Professor Everett's very valuable book of Units and Physical Constants the absolute results obtained by Mr. Macfarlane are given as the "results of experiments on the loss of heat from blackened and polished copper in air at atmospheric pressure," and no reference is made either to the shape or to the size of the cooling body.

[November 19, 1891.—Since this paper was sent in to the Royal Society, a new edition of this book has appeared, and, in consequence of a suggestion made to Professor Everett, the word "balls" has been added after the word "copper" in this new edition, as well as the following paragraph:—

" *Influence of Size.*

"According to Professor Ayrton, who quotes a table in 'Box on Heat,' the coefficient of emission increases as the size of the emitting body diminishes, and for a blackened sphere of radius $r$ cm. may be stated as

$$\text{“}\, 0\cdot0004928+\frac{0\cdot0003609}{r}.$$

"The value in M'Farlane's experiments was 2."]

The laws which govern the loss of heat from thin cylindrical conductors have not only considerable scientific interest in showing how the shape of a body affects the convection currents, but they are of especial importance to the electrical engineer in connexion with glow lamps, hot-wire voltmeters, fuses, &c. We, therefore, thought it desirable to ascertain the way in which the law of cooling for thick wires, which involved the diameter raised to the power three halves, passed into the law for the cooling of thin wires, involving only the first power of the diameter. For this object, the investigation described in the paper was commenced at the beginning of 1888, and the emissivity was measured of nine platinum wires, having the diameters of 1·2, 2·0, 2·9, 4·0, 6·0, 8·1, 9·3, 11·1, and 14 mils, or thousandths of an inch.

Suspecting that some of the published results concerning the currents required to fuse wires had been much influenced by the cooling action of the blocks to which the ends of the wires were attached, we started by making a calculation of the length necessary to give to our wires, so that the loss of heat by conduction should not introduce any important error into the determination of the emissivity. To do this it was necessary to calculate the distribution of temperature along a

wire through which a steady current was flowing and from which heat was lost by radiation, convection, and conduction, and it was further necessary to improve on the calculation one of us had published on this subject in the 'Electrician' for 1879, by taking into account the fact that the emissivity, as well as the thermal and electric conducting power, of the wire differed at different points in consequence of the difference of temperature.

Until we had completed the experiments described in this paper we could, of course, only employ in this calculation values that we had guessed at as being something near the truth for the emissivity of platinum wire for different diameters and at different temperatures. Hence, after the completion of the experiments, we took up the mathematical investigation again, substituting for the emissivity such a function of the diameter of the wire and the temperature of the point as we had experimentally found it to be. Section IV of the paper contains the investigation by which we finally arrived at the calculated distribution of temperature along the wire, and we have to express our sincere thanks to Professor Henrici (whom we consulted as to the best method of practically solving the rather complex differential equation arrived at) for the warm interest that he has taken in the mathematical treatment of the subject, and for the many suggestions which he has made, and which have enabled us to arrive at the mathematical solution given in the paper.

Each wire to be tested was stretched along the axis of a water jacketed cylinder 32·5 cm. long, the inner surface of which was blackened and kept at a constant temperature by a stream of water flowing through the jacket. The rate at which heat was lost by any one of the wires was measured by the product of the current passing through it into the P.D. (potential difference) maintained between its ends, while the ratio of the P.D. to the current gave the resistance of the wire and, therefore, its temperature. Experiments were in this way made with various currents flowing through each of the nine wires.

As the variation of resistance with temperature is known to vary with different specimens of platinum, experiments were separately made to determine the actual law of variation of resistance with temperature up to 300° C. for each piece of wire that had been employed in the emissivity experiments.

In this later determination various thermometers were used, and the subsequent comparison of these thermometers with a Kew standard thermometer involved a vast amount of labour, from the fact that it is, or at any rate was not possible three years ago, to purchase from the Kew Observatory a standard thermometer reading from, say, 200° to 300° C., with a short, wide chamber at the base in which the mercury expanded below 200° C. All that could be obtained was a

long thermometer which had been carefully tested between 0° and 100° C., and the remainder of whose tube had been simply calibrated for uniformity of bore. The consequence was that when we desired to compare one of our thermometers reading, say, from 200° to 300° C., with the Kew standard, their bulbs were very far apart when both were immersed in the oil-bath, and with the tops of the mercury columns just above the surface of the oil. A short description is given in the paper of the devices employed to overcome this difficulty and which enable an accurate comparison to be made between the thermometers.

On examining the curves accompanying the complete paper which show the emissivity for each temperature for each of the nine wires, we see that:—

1. For any given temperature the emissivity is the higher the finer the wire.

2. For each wire the emissivity increases with the temperature, and the rate of increase is the greater the finer the wire. For the finest wire the rate of increase of emissivity with temperature is very striking.

3. Hence the effect of surface on the total loss of heat (by radiation and convection) per second per square centimetre per 1° C. excess temperature increases as the temperature rises.

On comparing the loss of heat from the wire of 1·2 mils diameter when at 300° C. with that from the wire of 6 mils diameter when at 15° C., both being in an enclosure at 10° C., we see that the former loses per square centimetre of surface per second not

$$\frac{300-10}{15-10}, \text{ or } 58 \text{ times}$$

as much heat as the latter, as it would if the emissivity were the same; but, instead,

$$60 \times 58 \text{ or } 3480 \text{ times}$$

as much heat; arising from the fact that the emissivity, that is, the number of calories (gramme C.°) lost per second per square centimetre of surface per 1° C. excess temperature of the 1·2-mil wire at 300° C., is 60 times as great as that of the 6-mil wire at 15°, the latter varying very rapidly with the temperature near 15° C.

From the curves the following table (p. 170) has been drawn up, giving the emissivities of the various wires at eight useful temperatures.

We find that the emissivity of platinum wires of different diameters at the same temperature can be very fairly expressed by a constant *plus* a constant into the reciprocal of the diameter of the wire. For example, we find that

| Diameter of wire in | | Emissivities. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Mils. | Millimetres. | 40° C. | 60° C. | 80° C. | 100° C. | 150° C. | 200° C. | 250° C. | 300° C. |
| 1·2 | 0·0305 | 0·008230 | 0·009560 | 0·010300 | 0·010846 | 0·011875 | 0·012783 | 0·013625 | 0·014400 |
| 2·0 | 0·0508 | 0·005950 | 0·006860 | 0·007500 | 0·007900 | 0·008600 | 0·009070 | 0·009480 | 0·009850 |
| 2·9 | 0·0737 | 0·002193 | 0·003336 | 0·004086 | 0·004552 | 0·005095 | 0·005379 | 0·005628 | 0·005845 |
| 6·0 | 0·1524 | 0·002460 | 0·002660 | 0·002806 | 0·002930 | 0·003212 | 0·003460 | 0·003666 | 0·003837 |
| 8·1 | 0·2057 | — | — | — | 0·002804 | 0·002939 | 0·003076 | 0·003217 | 0·003352 |
| 9·3 | 0·2362 | — | — | — | 0·002297 | 0·002448 | 0·002586 | 0·002718 | 0·002843 |
| 11·1 | 0·2819 | — | — | — | 0·002053 | 0·002216 | 0·002363 | 0·002490 | 0·002608 |
| 14·0 | 0·3556 | — | — | — | 0·001894 | 0·002027 | 0·002136 | 0·002224 | 0·002286 |

The wire of 4 mils diameter is omitted from the table, as the experiments showed that its specific resistance was much greater, its temperature coefficient much smaller, and its emissivity much smaller than if it had been of platinum. This piece of wire probably therefore contained iridium or silver.

At 100° C. $e = 0{\cdot}0010360 + 0{\cdot}0120776d^{-1}$ .............. (1),
„ 200 „ $e = 0{\cdot}0011113 + 0{\cdot}0143028d^{-1}$ .............. (2),
„ 300 „ $e = 0{\cdot}0011353 + 0{\cdot}016084\ d^{-1}$ .............. (3),

where $d$ is the diameter of the wire in mils, or thousandths of an inch.

The emissivities have been calculated in calories lost per second per square *centimetre* per 1° C. excess temperature, in order that they may be compared with the emissivities obtained by other experimenters, but we prefer to speak of the diameters of the wires in *mils*, since a wire of 1 mil is about the finest that is drawn commercially. Hence the statement that the diameters of wires are 1, 2, or 3 mils is more suggestive to an engineer than saying that they are 0·0254, 0·0508, or 0·0762 millimetres.

The statement, not unfrequently made, that the current required to maintain a wire of a given material at a given temperature above that of the surrounding envelope is proportional to the diameter of the wire raised to the power three halves, is equivalent to stating that the emissivity is independent of the diameter. Now from the three formulæ (1), (2), (3), given above for $e$, we may conclude—

That for a temperature of 100° C. the value of $d$ in the formula

$$e = 0{\cdot}0010360 + 0{\cdot}0120776d^{-1}$$

must be something like 220 mils, or 5·6 mm., in order that the neglect of the second term may not make an error in $e$ of more than 5 per cent., and something like 1·15 inch, or 29·3 mm., if the error is not to exceed 1 per cent.;

That for a temperature of 200° C. the value of $d$ in the formula

$$e = 0{\cdot}0011113 + 0{\cdot}0143028d^{-1}$$

must be something like 244 mils, or 6·2 mm., in order that the neglect of the second term may not make an error in $e$ of more than 5 per cent., and something like 1·28 inches, or 32·5 mm. if the error is not to exceed 1 per cent.;

And that for a temperature of 300° C. the value of $d$ in the formula

$$e = 0{\cdot}0011353 + 0{\cdot}016084d^{-1}$$

must be something like 267 mils, or 6·8 mm., in order that the neglect of the second term may not make an error in $e$ of more than 5 per cent., and something like 1·39 inches, or 35·3 mm., if the error is not to exceed 1 per cent.

Generally, then, we may conclude that to assume that the emissivity is a constant for wires whose diameters vary from a small value up to 1 inch is to make a large error in the case of the greater number of

the wires, and an error of hundreds per cent. in the case of some of them.

Using the formula (3) which we have arrived at for determining the emissivity of platinum wires of different diameters at 300° C., it follows that to maintain a platinum wire 0·75 mil in diameter at 300° C. would require a current density of 331,000 amperes per square inch, and, if the emissivity of a copper wire of the same diameter and at the same temperature may be taken as being the same, it follows that to maintain a copper wire 0·75 mil in diameter at 300° C. would require a current density of 790,000 amperes per square inch.

## II. "On the Time-Relations of the Excursions of the Capillary Electrometer, with a Description of the Method of using it for the Investigation of Electrical Changes of Short Duration." By GEORGE J. BURCH, B.A. Oxon. Communicated by Professor BARTHOLOMEW PRICE, F.R.S. Received September 3, 1891.

(Abstract.)

This paper is in continuation of the author's preliminary note "On a Method of determining the Value of Rapid Variations of a Difference of Potential by means of the Capillary Electrometer," and describes a further simplification of the method then brought forward, consequent on a change in the mode of producing the photographic record of an excursion.

The rapidity of the movement of the meniscus was found to be affected by (1) the degree of concentration of the acid, (2) the length of the capillary beyond the end of the mercury column, (3) the shape of the tube where it tapers to form the capillary, (4) the shape of the orifice. These things might be taken as indicating the action of both mechanical friction and electrical resistance in determining the rate of movement. As was announced in the preliminary note, under ordinary circumstances the instrument is perfectly dead-beat; the meniscus commences to move the instant a difference of potential is communicated to the instrument, and stops directly it is withdrawn. The conditions under which overshooting may occur, and the possible extent of it, are discussed. It was found that, in general, the time-relations of the movement might be expressed by the equation

$$y = ae^{-ct},$$

in which $y$ is the distance of any point upon the curve from its asymptote. The tabular logarithms of a series of ordinates corre